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**COMMENT**

# Comment on 'Analytical results for a Bessel function times Legendre polynomials class integrals'

**V V Dodonov**

Instituto de Física, Universidade de Brasília, Caixa Postal 04455, 70910-900 Brasília, DF, Brazil

E-mail: vdodonov@fis.unb.br

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**Abstract**
I show that the result of the paper by Neves *et al* (2006 *J. Phys. A: Math. Gen.* **39** L293–6) is a special case of the well-known 'Gegenbauer finite integral'.

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The authors of recent paper [1] gave a detailed proof of the integral formula for the product of the associated Legendre polynomial and Bessel function:

$$\int_0^\pi \mathrm{d}\theta\,\sin\theta\,\exp(\mathrm{i}R\cos\alpha\cos\theta)P_n^m(\cos\theta)J_m(R\sin\alpha\sin\theta) = 2\mathrm{i}^{n-m}P_n^m(\cos\alpha)j_n(R), \tag{1}$$

where

$$j_n(z) = \sqrt{\frac{\pi}{2z}}J_{n+1/2}(z). \tag{2}$$

Their motivation was that 'this integral is not shown in any Integral Tables, nor in calculation packages such as Mathematica, and we do not know of any other report of this result' [1].

As a matter of fact, formula (1) is a special case of the formula containing the product of the Gegenbauer polynomial and Bessel function

$$\int_0^\pi \mathrm{d}\theta(\sin\theta)^{\nu+1/2}\exp(\mathrm{i}z\cos\psi\cos\theta)C_r^\nu(\cos\theta)J_{\nu-1/2}(z\sin\psi\sin\theta)$$
$$= \left(\frac{2\pi}{z}\right)^{1/2}\mathrm{i}^r(\sin\psi)^{\nu-1/2}C_r^\nu(\cos\psi)J_{\nu+r}(z) \tag{3}$$

if one takes into account the known relation (see, e.g., [2])

$$P_n^m(\cos\theta) = \frac{(2m)!}{m!2^m}(-\sin\theta)^m C_{n-m}^{m+1/2}(\cos\theta). \tag{4}$$

Formula (3) can be found in Watson's book [3] (equation (12.14.1) on page 379). Watson named it 'the Gegenbauer finite integral' and gave references to the original papers by

Gegenbauer of 1877 and 1882. It is given also in [4] (equation (7.8(12)) on page 57). The real and imaginary parts of the integral (3) can be found in [2] (equations (7.333.1) and (7.333.2)).

## References


[1] Neves A A R, Padilha L A, Fontes A, Rodriguez E, Cruz C H B, Barbosa L C and Cesar C L 2006 *J. Phys. A: Math. Gen.* **39** L293–6

[2] Gradshteyn I S and Ryzhik I M 1980 *Table of Integrals, Series, and Products* (New York: Academic)

[3] Watson G N 1962 *A Treatise on the Theory of Bessel Functions* (Cambridge: Cambridge University Press)

[4] Erdélyi A (ed) 1953 *Bateman Manuscript Project: Higher Transcendental Functions* vol II (New York: McGraw-Hill)